ESM 4A Project
Is the one euro coin biased?

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Abstract

In this paper the bias of a German one euro coin is investigated. Two different German one euro coins have been tossed 1000 times each. Obtained results have been tested at significance levels of 5% and 1% to determine if the coin is biased.

Contents

1 Introduction

In this paper the fairness/bias of German one euro coin with respect to tossing is examined.

This topic is of particular interest because what does one do if he or she cannot decide? One of the common solutions is to toss a coin. Usually everyone expects the coin to be fair and hence the outcome to be truly random. Therefore tossing a coin is used to make decisions in many areas including sports (who chooses side/gets ball first in a soccer match), room allocation (in case more people apply for the same room) and many others. Such a decision is then accepted as a fair decision. Therefore using a biased coin could be used to turn the decision into one's favor while still having it accepted as a fair decision, so it is of particular interest to know if a coin is biased.

An article in The Guardian [3] from January 4, 2002 reported that the Belgian one euro coin was found to be biased. This conclusion was made by two Polish Mathematicians based on an experiment obtaining 140 heads in 250 tosses. I also have found articles/reports claiming the French one euro coin to be biased. Since the euro coins have a common tail (back side) and a different (in each country) head (front side), bias by design seems possible. However, most of the articles/reports used a relatively low (not significant) number of tosses to draw their conclusions or did not even state the number of times they tossed their coin. Since I failed to find comprehensive data on tossing the one euro coin I have decided to find out on my own if the coin really is biased or not.

(a) head [1]　　(b) tail [2]

Figure 1: German one euro coin

In particular I am interested in bias caused by design of the coin, rather than some artificial post-production biasing such as dirt, physical damage to the coin changing its shape or other types of bias not caused by design. For several reasons I have decided to test only the German one euro coin. First of all I found no data on the German coin, all I found was mostly about the Belgian one. Furthermore, since I was doing the experiment in Germany, it was much easier to obtain German coins rather than other countries' coins.

In this way I could have chosen from more coins and pick the most suitable ones - cleanest (minimize bias caused by inhomogeneous distribution of dirt on the coin) and least damaged coins so that the coins would be biased only by design.

2 Data

Since I found no reasonable data on the German one euro coin, I have collected the data on my own. Two one euro German coins have been tossed, each 1000 times. One of the coins has been tossed by me while the other one was tossed by my girlfriend. In this way I hoped to minimize bias caused by the particular way of tossing the coin (different initial velocity of the coin upwards and different initial angular velocity of the rotations, or rather slightly different distributions of these quantities). While one of us was tossing the coin, the other one was collecting the data. This way the data collection process was faster and less error-prone as one was checking if the other entered the correct data. For the purpose of counting the raw data a simple C program was used to further minimize human caused errors.

Although it might seem clear and straightforward how to toss a coin, there are in general three ways of tossing/spinning a coin:

1. spinning on a desk and waiting until the coin stops spinning and falls on one of its sides
2. tossing in the air and letting the coin land on some surface like ground or desk. A softer surface (like textile) might be used to damp the landing and prevent jumping off the surface after the first landing.
3. tossing in the air and catching the coin with your hand while it is rotating in the air. After catching it, it is placed onto the other hand and the top side of the coin is the one which counts.

I have decided to use the third way of tossing because it is the one which is most used in practice and easiest to do. If the coin did not reach the height of at least 20 cm or was rotating too slow (making less than 2 rotations in the air) then the toss was not included in the results and the coin had to be tossed again. Furthermore, the coin was always held heads up before being tossed to achieve the same initial conditions for each toss.

Coin #1

Heads:	497
Tails:	503
Total:	1000

Coin #2

Heads:	494
Tails:	506
Total:	1000

The tail and head of a one euro coin are referred to as shown in figure 1.

The collected data is *significant* because the error of sampling for each coin is $\frac{1}{\sqrt{1000}} \approx 0.0316$ which is acceptable for the purpose of my investigation.

The data is *representative* because two (rather than one) coins have been tossed. These coins have been chosen in such a way as to have no post-production bias (dirt, physical damage, ...). Besides that the coins were tossed by two different people, so if there were bias caused by a certain way of tossing, it should show up as a difference between the two coins.

3 Evaluation

The null hypothesis is that *the coin is fair (not biased).* The data (each coin separately) will be tested against the null hypothesis using two different tests at significance levels of 5% and 1%.

test1

To test the null hypothesis collected data will be compared with the expected binomial distribution $B(n,p)$ with parameters $p = q = 0.5$ and $n = 1000$. To calculate the binomial distribution values, factorials of large numbers would have to be evaluated. This is possible, but not feasible. A computationally more feasible solution is to approximate the binomial distribution by a normal distribution $N(np, \sqrt{npq})$ where
$\mu = np = 500$
$\sigma = \sqrt{npq} = \sqrt{250} \approx 15.811$

Let x be the observed number of heads, X stand for the observed distribution and Z for the normalized distribution. We want to calculate x_1, x_2 such that $P(x_1 \leq X \leq x_2) = 0.95$.

For 5% significance level:
$P(X \leq x_2) = 0.975$
$P(Z \leq z_2) = 0.975$
$z_2 = 1.96$ (using the tables for normal distribution)
$x_2 = z_2 \cdot \sigma + \mu$
$x_2 = 1.96 \cdot 15.811 + 500 \approx 531$
$P(x_1 \leq X) = 0.025$
$P(z_1 \leq Z) = 0.025$
$P(Z \leq -z_1) = 0.975$
$z_1 = -1.96$ (using the tables for normal distribution) $x_1 = z_1 \cdot \sigma + \mu$
$x_1 = -1.96 \cdot 15.811 + 500 \approx 469$
Hence we would reject the null hypothesis if the observed number of heads

were not within $469 \le x \le 531$. Since $x = 497$ for coin #1 and $x = 494$ for coin #2, we cannot reject the null hypothesis at the 5% significance level for any of the coins.

For 1% significance level:
$P(X \le x_2) = 0.995$
$P(Z \le z_2) = 0.995$
$z_2 = 2.575$ (using the tables for normal distribution)
$x_2 = z_2 \cdot \sigma + \mu$
$x_2 = 2.575 \cdot 15.811 + 500 \approx 541$
$P(x_1 \le X) = 0.005$
$P(z_1 \le Z) = 0.005$
$P(Z \le -z_1) = 0.995$
$z_1 = -2.575$ (using the tables for normal distribution) $x_1 = z_1 \cdot \sigma + \mu$
$x_1 = -2.575 \cdot 15.811 + 500 \approx 459$

Hence we would reject the null hypothesis if the observed number of heads were not within $459 \le x \le 541$. Since $x = 497$ for coin #1 and $x = 494$ for coin #2, we cannot reject the null hypothesis at the 1% significance level for any of the coins.

test2

Next, the χ^2 statistical test will be used to verify test the null hypothesis.

Coin #1:

	Observed (O_i)	Expected (E_i)	$\chi^2 = \frac{(O_i - E_i)^2}{E_i}$
Heads	497	500	$\frac{9}{500}$
Tails	503	500	$\frac{9}{500}$
Total			$\frac{18}{500} = 0.036$

Coin #2:

	Observed (O_i)	Expected (E_i)	$\chi^2 = \frac{(O_i - E_i)^2}{E_i}$
Heads	494	500	$\frac{36}{500}$
Tails	506	500	$\frac{36}{500}$
Total			$\frac{72}{500} = 0.144$

For both coins there are 2 bins and one constraint - the total number of trials. Hence we have $\nu = 2 - 1 = 1$ degree of freedom.

The theoretical χ^2 value is $\chi_1^2(0.95) = 3.841$ for 5% significance level and $\chi_1^2(0.99) = 6.635$ for 1% significance level. Obviously, both coins have their χ^2 below the theoretical χ^2 and hence at both singnificance levels the null hypothesis cannot be rejected for any of the coins.

Looking at the other end of the χ^2 distribution we see that $\chi_1^2(0.05) = 0.001$ and $\chi_1^2(0.01) = 0.0002$, so our data ($\chi^2 = 0.036$ and $\chi^2 = 0.144$) is not too good (i.e. it is not missing the expected randomness). The drawback of this χ^2 test is that there is just one degree of freedom, which makes the test rather unreliable.

The null hypothesis could not have been rejected for any of the two tested coins at neither 5% nor 1% significance levels using both tests and hence we conclude that the *German one euro coin is NOT biased.*

Error of the sample (data) is $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{1000}} \approx 0.0316$. This value is rather low and obviously does not have any impact on the results obtained in the two tests.

4 Conclusion

Two German one euro coins have been tossed 1000 times each and the obtained data has been tested against the null hypothesis of being fair at significance levels of 5% and 1% by comparing with the expected binomial distribution (actually approximated by normal distribution) as well as using the χ^2 statistical test. For none of the two tested coins could the null hypothesis be rejected. Therefore, it can be concluded that the German one euro coin is not biased.

Since the German one euro coin was not found to be biased, but the Belgian one Euro as well as French one Euro coins have been reported to be biased, it would be interesting to test with a significant number of tosses all the one Euro coins and see if/how many of them are biased. If some of them are biased while others not, it might be caused by the different national sides of the coins (tails).

References

[1] `http://www.euro.ecb.int/en/section/euro0/coins.CoinPar.Singlecd1.CommonFaceImage.gif`
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[2] `http://www.euro.ecb.int/en/section/euro0/specific.CountryCoinPar.SingleDECoincd1.SpecificFaceImage.gif`
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[3] `http://www.guardian.co.uk/euro/story/0,11306,627496,00.html`
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